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FREE AND FORCED CONVECTION FROM A ROTATING CONE

By Jay Fox

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio

INTRODUCTION

Simultaneous forced and free convection by laminar flow on an inverted cone rotating about a vertical axis in a quiescent fluid has been analyzed by Hering and Grosh (ref. 1). In their study, the expansion of the heated fluid produced a buoyancy force that was directed upward and away from the apex at the lowest point on the cone. A simultaneous swirling motion or the fluid was induced by the adherence of the fluid at the surface to the rotating cone. The surface temperature increased linearly with distance from the apex so that similar velocity and temperature profiles were obtained. In practice, however, the isothermal cone is most often encountered. Kreith and Kneisel (ref. 2) reported average heat-transfer performance on rotating isothermal cones with free and forced convection. Local heat-transfer results are difficult to obtain from experimentally determined average results especially in the present case because similar velocity and temperature profiles do not occur.

In the present study, a series solution to the isothermal problem is formulated, and the first terms of the series are presented for a Prandtl number of 0.72. From these results, local heat transfer and shear performance can be calculated.

ANALYSIS

Laminar boundary-layer equations suitable for a rotating cone with buoyancy forces in the axial direction are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{u}{x} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} - \frac{w^2}{x} = \beta g(\cos\,\gamma)(T - T_\infty) + \nu\,\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\,\frac{\partial w}{\partial x} + v\,\frac{\partial w}{\partial y} + \frac{uw}{x} = \nu\,\frac{\partial^2 w}{\partial y^2} \tag{3}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where x, y, and z are distances along the cone generators from the apex, normal to the cone, and along the circumference of the circular sections of the cone, respectively. The corresponding velocity components are u, v, and w. The standard property symbols α, β, and ν designate thermal diffusivity, expansion coefficient, and kinematic viscosity. The symbols T, g, and γ represent temperature, gravitational acceleration, and half the cone apex angle. In eqs. (1) to (4), no changes in the z-direction are shown since the flow is symmetrical about the cone axis. Buoyancy is represented by the usual free-convection term in the x-direction but has been neglected in the y-direction.

The stream function, defined by

$$xu = \frac{\partial\psi}{\partial y},\ xv = -\,\frac{\partial\psi}{\partial x}$$

and the transformations

$$\psi = x^2(\nu\omega\,\sin\,\gamma)^{1/2}f(\eta)$$

$$w = x\omega(\sin\,\gamma)g(\eta)$$

$$T - T_\infty = (T_W - T_\infty)\theta(\eta)$$

$$\eta = y[\omega(\sin\,\gamma)/\nu]^{1/2}$$

where ω is the rotational speed of the cone and T_W is the cone temperature, are substituted into eqs. (1) to (4), and a series solution to the resulting equations is proposed:

$$f = f_0 + (Gr/Re^2)f_1 + \cdots \tag{5a}$$

$$g = g_0 + (Gr/Re^2)g_1 + \cdots \tag{5b}$$

$$\theta = \theta_0 + (Gr/Re^2)\theta_1 + \cdots \tag{5c}$$

where $Gr = \beta g\,(T_w - T_\infty)x^3(\cos\gamma)/\nu^2$ and $Re = x^2\omega(\sin\gamma)/\nu$ are the Grashof and the Reynolds numbers. Positive or negative values of Gr/Re^2 are associated with buoyancy forces that aid or retard the forced flow away from the cone apex. Two sets of ordinary differential equations result from equation of like powers of Gr/Re^2:

$$f_0''' + 2f_0f_0'' - f_0'^2 + g_0^2 = 0 \tag{6a}$$

$$g_0'' + 2f_0g_0' - 2f_0'g_0 = 0 \tag{6b}$$

$$\theta_0'' + 2\ Pr\ f_0\theta_0' = 0 \tag{6c}$$

$$f_1''' + 2f_0f_1'' + 2f_1f_0'' - 2f_0'f_1' + 2g_0g_1 + \theta_0 = 0 \tag{7a}$$

$$g_1'' + 2f_0g_1' + 2f_1g_0' - 2g_0f_1' - 2g_1f_0' = 0 \tag{7b}$$

$$\theta_1'' + 2\ Pr(f_0\theta_1' + f_1\theta_0') = 0 \tag{7c}$$

where $Pr = \nu/\alpha$ is the Prandtl number. Corresponding to the physical boundary conditions $u = v = 0$, $w = x\omega\sin\gamma$, $T = T_w$ at $y = 0$, and $u \to v \to w \to (T - T_\infty) \to 0$ as $y \to \infty$, the transformed conditions are

$$\left.\begin{array}{l} \left.\begin{array}{l} f_0 = f_0' = 0,\ g_0 = 1,\ \theta_0 = 1 \\ f_1 = f_1' = g_1 = \theta_1 = 0 \end{array}\right\}\eta = 0 \\ \left.\begin{array}{l} f_0' \to g_0 \to \theta_0 \to 0 \\ f_1' \to g_1 \to \theta_1 \to 0 \end{array}\right\}\eta \to \infty \end{array}\right\} \tag{8}$$

Eqs. (6) and (8) are the same as those for the rotating cone or disk without buoyancy that have been numerically integrated by Ostrach and Thornton (ref. 3), among others. The tabulated results of ref. 3 provided the necessary starting information for the numerical integration of eqs. (7) and (8). Functions f_1, g_1, and θ_1 for a Prandtl number of 0.72 are shown in fig. 1 together with f_0, g_0, and θ_0. Surface derivatives are listed in table I.

TABLE I. - SURFACE DERIVATIVES OF TEMPERATURE AND VELOCITY

$f_0''(0)$	$g_0'(0)$	$\theta_0'(0)$	$f_1''(0)$	$g_1'(0)$	$\theta_1'(0)$
0.5102	-0.6159	-0.3286	0.6200	-0.5044	-0.4002

RESULTS

Changes in local surface shear (in x- and y-directions) and heat transfer due to buoyancy can be visualized by forming ratios

$$\tau_x/\tau_{x0} = f''(0)/f_0''(0) = 1 + (Gr/Re^2)f_1''(0)/f_0''(0) + \cdots$$

$$= 1 + 1.215(Gr/Re^2) + \cdots \qquad (9a)$$

$$\tau_z/\tau_{z0} = g'(0)/g_0'(0) = 1 + (Gr/Re^2)g_1'(0)/g_0'(0) + \cdots$$

$$= 1 + 0.8189(Gr/Re^2) + \cdots \qquad (9b)$$

$$q/q_0 = \theta'(0)/\theta_0'(0) = 1 + (Gr/Re^2)\theta_1'(0)/\theta_0'(0) + \cdots$$

$$= 1 + 1.218(Gr/Re^2) + \cdots \qquad (9c)$$

where 0 subscripts refer to quantities on a rotating cone without buoyancy effects that can be calculated from the definitions of shear and heat flux:

$$\tau_x = \mu(\partial u/\partial y)_{y=0} = \mu x(\omega \sin \gamma)^{3/2}f''(0)/\nu^{1/2} \qquad (10a)$$

$$\tau_z = \mu(\partial w/\partial y)_{y=0} = \mu x(\omega \sin \gamma)^{3/2}g'(0)/\nu^{1/2} \qquad (10b)$$

$$q = -k(\partial T/\partial y)_{y=0} = -k(T_w - T_\infty)(\omega \sin \gamma)^{1/2}\theta'(0)/\nu \qquad (10c)$$

and the values from ref. 3 that are listed in table I. The shear and heat flux ratios defined by eq. (9) are displayed in fig. 2 along with the corresponding results from the similarity solutions of ref. 1.

Calculations in ref. 1 were carried out only for flows with the buoyancy forces directed away from the cone apex corresponding to $(Gr/Re^2) > 0$ in fig. 2. The isothermal solution is valid for flows with buoyancy directed either toward or away from the cone apex, as shown by the negative and positive abscissa values in fig. 2.

The increase of τ_x/τ_{x0} with Gr/Re^2 in the similarity case is nearly linear, while the other ratios in fig. 2 increase with more curvature. If this trend carries over into the isothermal solution, the truncated series for τ_x is a better approximation to the final solution than those for τ_z and q since curvature is introduced by higher order terms than those presented herein.

The accuracy of the isothermal solution is limited by the nature of series solutions to small values of the expansion parameter Gr/Re^2. Since $Gr/Re^2 \sim 1/x$, the solution improves in accuracy with x. In fact, the convergence of the solution can only be expected away from the cone apex. Actually, this feature is an advantage since the boundary-layer idealizations do not describe the flow in the neighborhood of the cone apex but are a good approximation of the flow in the region of convergence of the series solution.

REFERENCES

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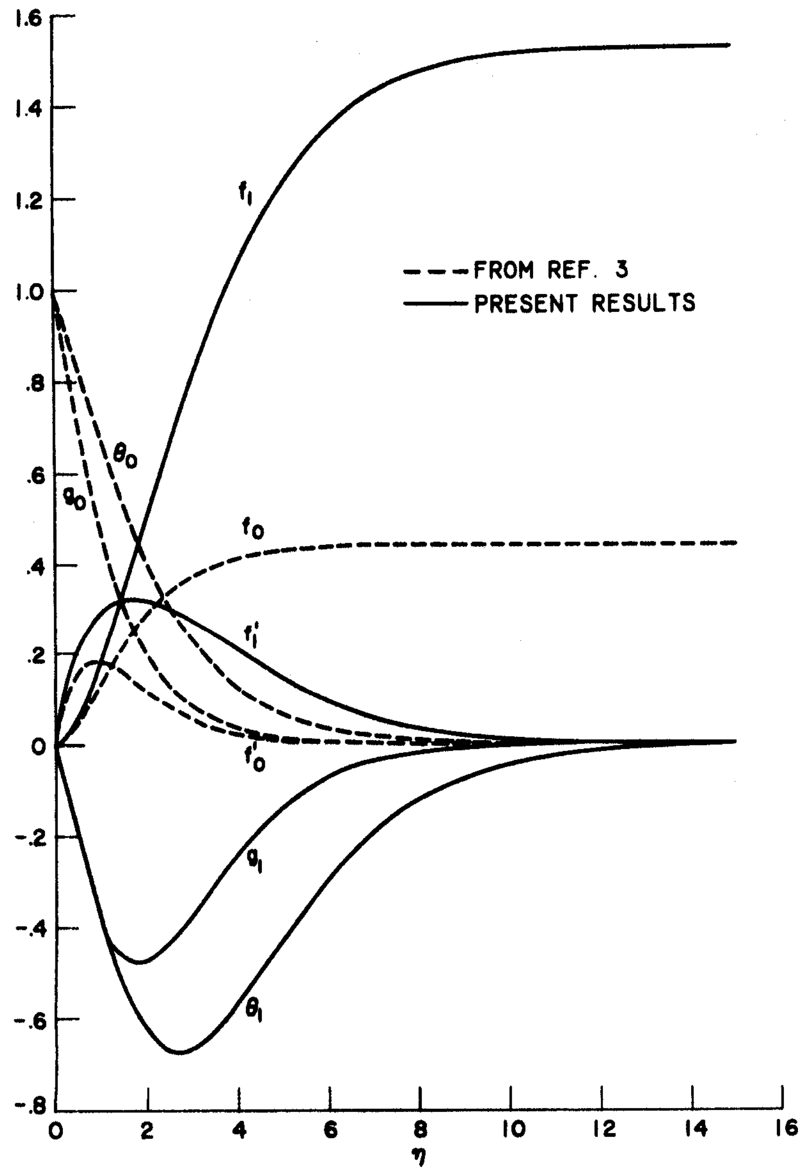

Fig. 1. - Velocity and temperature functions.

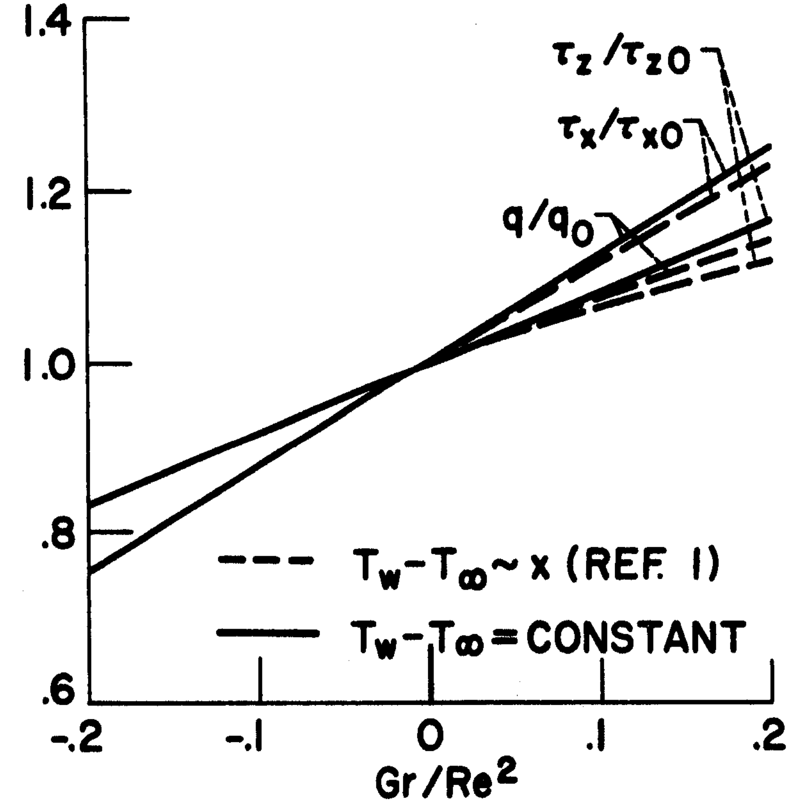

Fig. 2. - Effect of buoyancy on surface shear and heat transfer.

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